November 02, 2017

Calculus Warm Up #9-4

To differentiate when the base is not e...
1) Take the ln of both sides, bring exponent in front
2) d/dx both sides
3) Multiply both sides by y
4) Replace y with original expression. Ta dah!

You try:

$y = 3^x$ $y = 7^x$

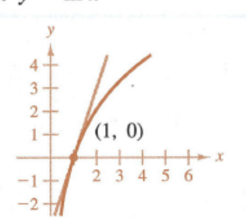
What do you notice about the pattern?

Generalize for $y = a^x$

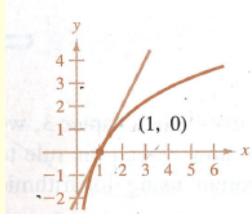
HW Questions: p. 392 Whoops! Save #25-37 for tonight!

In Exercises 1–4, find the slope of the tangent line to the given logarithmic function at the point (1, 0).

1. $y = \ln x^3$



3. $y = \ln x^2$



In Exercises 5–38, find dy/dx.

5. $y = \ln x^2$

7. $y = \ln \sqrt{x^4 - 4x}$

9. $y = (\ln x)^4$

11. $y = \ln(x\sqrt{x^2 - 1})$

13. $y = \ln\left(\frac{x}{x^2 + 1}\right)$

15. $y = \frac{\ln x}{x^2}$

17. $y = \ln(\ln x^2)$ $\longrightarrow$ $y = \ln u$ $\quad u = \ln x^2$

19. $y = \ln\sqrt{\frac{x+1}{x-1}}$

21. $y = \ln\left(\frac{\sqrt{4 + x^2}}{x}\right)$

$y' = \frac{1}{\ln x^2} \cdot \frac{2}{x}$ $\quad \frac{du}{dx} = \frac{2}{x}$

$= \frac{2}{x \ln x^2}$

$= \frac{1}{x \ln x}$

23. $y = \dfrac{-\sqrt{x^2+1}}{x} + \ln\left(x + \sqrt{x^2+1}\right)$

$$y = \frac{-(x^2+1)^{1/2}}{x} + \ln\left(x + (x^2+1)^{1/2}\right)$$

$$y' = \frac{1}{x^2\sqrt{x^2+1}} + \frac{1}{x+\sqrt{x^2+1}} \cdot \left(1 + \frac{1}{\cancel{2}}(x^2+1)^{-1/2}(\cancel{2}x)\right)$$

$$= \frac{1}{x^2\sqrt{x^2+1}} + \frac{\left(1 + x(x^2+1)^{-1/2}\right)}{x+\sqrt{x^2+1}} \cdot \frac{\left(x-(x^2+1)^{1/2}\right)}{x-\sqrt{x^2+1}}$$

$$= \frac{1}{x^2\sqrt{x^2+1}} + \frac{\boxed{\cancel{x}} - (x^2+1)^{1/2} + x^2(x^2+1)^{-1/2} \boxed{\cancel{-x(x^2+1)^0}}}{x^2-(x^2+1)}$$

$$= \frac{1}{x^2\sqrt{x^2+1}} + \frac{x^2(x^2+1)^{-1/2} - (x^2+1)^{1/2}}{-1}$$

$$= \frac{1}{x^2\sqrt{x^2+1}} - (x^2+1)^{-1/2}\left[x^2 - (x^2+1)^1\right]$$

$$\frac{1}{x^2\sqrt{x^2+1}} + \frac{x^2 1}{x^2\sqrt{x^2+1}}$$

$$\frac{(1+x^2)^1}{x^2(x^2+1)^{1/2}} \rightarrow \frac{\sqrt{x^2+1}}{x^2}$$

7.5 Day 2: Logarithmic Differentiation

<u>Steps:</u> 1. Take the natural log of both sides

2. Use log properties to expand/simplify

3. Implicitly differentiate ($\frac{d}{dx}$ both sides)

4. Solve for y'

5. Replace y with your original expression and simplify.

Try it! $y = x\sqrt{x^2+1}$

$y = x\sqrt{x^2 + 1}$

$\ln y = \ln\left(x\sqrt{x^2+1}\right)$

$\ln y = \ln x + \frac{1}{2}\ln(x^2+1)$

$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{x} + \frac{\cancel{2}x}{\cancel{2}(x^2+1)} \cdot \frac{x}{x}$

$y \cdot \frac{1}{y}\frac{dy}{dx} = \frac{x^2+1+x^2}{\cancel{x}(x^2+1)^1} \cdot \frac{\cancel{x}(x^2+1)^{1/2}}{1}$

$= \frac{(2x^2+1)}{\sqrt{x^2+1}}$

Find the derivative of

$$y = \frac{(x-2)^2}{\sqrt{x^2+1}}.$$

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$$y = \frac{(x-2)^2}{\sqrt{x^2+1}}.$$

$$\ln y = 2\ln(x-2) - \frac{1}{2}\ln(x^2+1)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{(x-2)} - \frac{\cancel{2}x}{\cancel{2}(x^2+1)} \cdot \frac{(x-2)}{(x-2)}$$

$$y \cdot \frac{1}{y}\frac{dy}{dx} = \frac{2(x^2+1) - x^2+2x}{(x-2)(x^2+1)} \cdot y$$

$$\frac{dy}{dx} = \frac{x^2+2x+2}{\cancel{(x-2)}(x^2+1)} \cdot \frac{(x-2)^{2}}{(x^2+1)^{1/2}}$$

$$= \frac{(x-2)(x^2+2x+2)}{(x^2+1)^{3/2}}$$

Rule #1 $\frac{d}{dx}[a^x] = (\ln a)a^x$ ← From the warm up

Rule #2 $\frac{d}{dx}[a^u] = (\ln a)a^u \frac{du}{dx}$

Rule #3 $\frac{d}{dx}[\log_a x] =$

Let $y = \log_a x$

$$\ln a^y = \ln x$$

$$\frac{d}{dx}\left[y(\ln a) = \ln x\right]$$

$$(\ln a)\frac{dy}{dx} = \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{1}{(\ln a)x}$$

Rule #1 $\frac{d}{dx}[a^x] = (\ln a)a^x$

Rule #2 $\frac{d}{dx}[a^u] = (\ln a)a^u \frac{du}{dx}$

Rule #3 $\frac{d}{dx}[\log_a x] = \frac{1}{(\ln a)x}$

Rule #4 $\frac{d}{dx}[\log_a u] = \frac{1}{(\ln a)u} \frac{du}{dx}$

Find the derivatives of the following.

(a) $y = 2^x$ (b) $y = \log_2 (x^2 + 1)$

$\ln y = x \ln 2$

$y \cdot \frac{1}{y} \frac{dy}{dx} = (\ln 2)(2)^x$

$2^y = x^2 + 1$

$y(\ln 2) = \ln(x^2 + 1)$

$\frac{dy}{dx} \ln 2 = \frac{1(2x)}{x^2+1}$

$\frac{dy}{dx} = \frac{2x}{(\ln 2)(x^2+1)}$

EXAMPLE 10 Comparing variables and constants

(a) $\frac{d}{dx}[e^e] = 0$ Constant Rule

(b) $\frac{d}{dx}[e^x] = e^x$ Exponential Rule

(c) $\frac{d}{dx}[x^e] = ex^{e-1}$ Power Rule

(d) $y = x^x$ Logarithmic differentiation

$$\frac{d}{dx}\left[\ln y = (x)(\ln x)\right]$$

$$\frac{1}{y}\frac{dy}{dx} = x \cdot \frac{1}{x} + (1)\ln x$$

$$y \cdot \frac{1}{y}\frac{dy}{dx} = (1 + \ln x)(x^x)$$

$$\frac{dy}{dx} = x^x(1 + \ln x)$$

HW: p. 392

25 - 59 odd

Group Quiz tomorrow: 7.2